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Langmuir Probe Response in a Turbulent Plasma

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A theoretical and experimental investigation of the response of collisionless cylindrical Langmuir probes in unsteady plasmas is made with the specific aim of developing methods for measuring the mean and statistical plasma properties. A comprehensive theory describing the transient responses of both single-probes and symmetric double-probes to arbitrary (low-frequency) fluctuations in electron density, plasma potential, electron temperature, and ion temperature is formulated by perturbing the probe steady-state equations about the mean plasma properties. The ratios of probe radius to debye length and applied potential to electron temperature are used as controllable parameters to form a set of equations from which the mean and rms properties may be determined. Experiments are performed in an unsteady highly expanded low-density flowing argon plasma. Pertinent features of the theory are verified and quantitative measurements of the mean and fluctuating plasma properties are made.

Nomenclature

A = probe area
E_N, E_Θ = normalized partial derivatives appearing in the equations defining the floating potential measurement
F_N, F_Θ = normalized partial derivatives appearing in the equations defining the single-probe current
G_N, G_Θ = normalized partial derivatives appearing in the equations defining the double-probe current
H,h = current in double-probe circuit
I = retarding field electron current
J,j = attractive field probe current (ion or electron)
K = defined by Eq. (5)
L = probe length
M = mass of species
N,n = charged particle density
q = one electronic charge
r = probe radius
t = time
U = freestream velocity
V = applied probe to probe or probe to reference potential
α = defined by Eq. (3)
β = defined by Eq. (4)
Θ,θ = temperature (eV)
λ = $[\Theta/(4\pi Nq)]^{1/2}$, electron debye length
τ = defined by Eq. (8)
Φ,ϕ = potential measured with respect to the plasma potential
χ = Φ/Θ nondimensionalized potential
ψ = V/Θ nondimensionalized potential

Subscripts

$+,-$ = positive ion, electron
1,2 = numerical designation for double-probes
f = open circuit potential
m = measured quantity
o = computed using mean plasma properties
p = plasma
w = probe surface

Superscripts

$'$ = ion current to a probe that is common to a double-probe system
$-$ = denotes time average

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1. Introduction

FLUCTUATIONS or instabilities are an important characteristic of many interesting plasmas. Because the free-molecule Langmuir probe has proved successful in measuring the local properties of steady plasmas, attempts have been made to use these probes to measure the mean and statistical properties of unsteady plasmas. Considerable work in this direction has been done where the earlier authors[1–4] attempted to predict the average probe current for fluctuations in electron density, plasma potential, and electron temperature. These analyses were based on the steady-state theories of Bohm[5] and of Langmuir and Mott-Smith,[6] thus restricting the attractive-field probe current collection (ion or electron) to the "thin sheath limit" (TSL) or the "orbital motion limit" (OML). Demetriades and Doughman[7] gave attention to the problem of measuring the actual magnitudes of the fluctuations. Beginning with the steady-state theory of Langmuir and Mott-Smith,[6] they found the influence of fluctuations in electron density, plasma potential, and electron temperature on the electron current collected by plane, cylindrical, and spherical probes operating with a large positive or negative bias. Because a closed set of equations usually could not be formed from these results, the mean and statistical properties were in general indeterminable. However, one notable exception was that the average current to a cylindrical probe operating with a large positive bias in the OML was independent of all fluctuations, while the rms current was a direct measure of the fluctuations in electron density. Recently Sutton[8] examined the possibility of using positively biased cylindrical Langmuir probes, with small r/λ, to measure the electron density fluctuations in hypersonic turbulent wakes. However, applications of the aforementioned results are quite limited by the restriction to current collection in the OML or TSL, since for many plasmas physical limitations associated with probe construction render the first limit $[(r/\lambda) < 1]$ unattainable while the assumption of collisionless collection eliminates the latter $[(r/\lambda) \gg 1]$. In addition, the problem of measuring the mean and statistical properties of plasmas characterized by separate ion and electron temperatures was not dealt with in these earlier investigations.

The purpose of the present paper is therefore to determine the response of cylindrical Langmuir probes in an unsteady plasma with the aim of developing methods for measuring the mean and statistical plasma properties. The influence of arbitrary (low-frequency) fluctuations in charged particle density, electron temperature, plasma potential, and ion temperature on the response of both single-probes and double-probes operating at moderate values of the applied potential in plasmas where the ratio of ion to electron temperature is less than one are considered, for a wide range of ratios of probe radius to debye length. A comprehensive theory describing the transient responses of both single-probes and symmetric double-probes is formulated by perturbing the collisionless probe steady-state equations about the mean plasma properties. Equations are derived which relate the instantaneous, average, and mean-square measured probe signals to the instantaneous, average, and mean-square plasma properties. These formulas are then used to determine the values of the correlation terms to be used in the correct deduction of the mean plasma properties using the equations relating the mean-measured properties to the actual mean plasma properties and their respective correlation terms. In turn, these mean properties may be used to improve the computation of the fluctuating magnitudes, and so on. The problem as it appears in this nonlinear form invites an iterative solution which converges rapidly when the fluctuations are small. Following the analysis possible simplifications and solutions are discussed and direct comparisons are made with experiment.

2. Steady-State Probe Responses

Several[9–11] good reviews of Langmuir probe theory and application are available in the literature. To date the most realistic collisionless probe theory has been given by Laframboise[12] who numerically solved the theoretical analysis previously formulated by Bernstein and Rabinowitz.[13] The results are valid for a wide range of ratios of probe radius to debye length and a complete range of ion to electron temperature ratios. Ample experimental confirmations of these results have been obtained in a weakly ionized flowing argon plasma[14,15] and in a low-density potassium plasma.[16] Therefore algebraic representations of the numerical results of Laframboise[12] are used to describe the cylindrical probe steady-state response. These relations are summarized here for completeness. The approximate analytical expressions derived by Kiel[17] which describe the probe current collection were not available at the time this analysis was performed.

Peterson[18] has shown that the magnitude of the current (ion or electron) attracted to a single-probe operating in a plasma consisting of electrons and singly charged ions may be represented by

$$J_{+,-} = K_{+,-} N\Theta_-^{1/2} (\beta + |\chi|)^\alpha \tag{1}$$

where

$$\chi = \Phi/\Theta_- \tag{2}$$

$$\alpha = a/[\ln(r/\lambda) + b] + c(\Theta_+/\Theta_-)^m + d \tag{3}$$

$$\beta = e + \{f + g[\ln(r/\lambda)]^3 - [l/\ln(r/\lambda)]\} \times (\Theta_+/\Theta_-) + \{l/[\ln(r/\lambda)]\} \tag{4}$$

$$K_{+,-} = 2\pi r L q(q/2\pi M_{+,-})^{1/2} \tag{5}$$

and a–m are numerical constants whose magnitudes as shown in Table 1 depend only on the species being collected. As shown in Ref. 18, values of probe current calculated using Eqs. (1–4) differ by <3% from the numerically computed values given by Laframboise,[12] provided $5 \leq r/\lambda \leq 100$, and $\chi > 10$ (<-3) for electron (ion) collection. In the electron-retarding field the electron current is assumed to be defined by the well-known expression

$$I = K_- N\Theta_-^{1/2} \exp(\chi) \tag{6}$$

where

$$\chi = \Phi/\Theta_- \equiv \chi_w - \chi_p \tag{7}$$

Therefore the potential at which the single-probe collects zero net current is

$$\Phi_f = \tau\Theta_- + \Theta_-\alpha \ln[\beta + |\chi_f|] = \Phi_{wf} - \Phi_p$$

where

$$\tau = -\tfrac{1}{2}\ln(M_+/M_-) \tag{8}$$

The electron temperature measurement from the single-probe characteristic is defined by the well-known relation

$$\Theta_- = [\partial(\ln I)/\partial\Phi]^{-1} \tag{9}$$

and once the electron temperature is known the charged particle density may be computed according to

$$N = [J_{+,-}(\chi)]/[K_{+,-}\Theta_-^{1/2}(\beta + |\chi|)^\alpha] \tag{10}$$

The current flowing in a double-probe circuit consisting of probes (1 and 2) of equal radii but with areas A_1 and A_2 was shown[18] to be

$$H = [J_{2+} - J_{1+}(A_2/A_1)\exp\psi]/[1 + (A_2/A_1)\exp\psi] \tag{11}$$

where

$$\psi = V/\Theta_- = \chi_2 - \chi_1,\ J_{2+} = J_{2+}(\chi_2),\ J_{1+} = J_{1+}(\chi_1)$$

The electron temperature measurement from the double-probe characteristic proceeds according to the relation

$$\Theta_- = \{(J_{1+}J_{2+}/[J_{1+} + J_{2+}])(\partial H/\partial V)^{-1} \times (1 + [\alpha/(\beta + |\chi|)])\}_{V=0} \tag{12}$$

Table 1 Numerical values of the constants appearing in Eqs. (3) and (4)

Constant	a	b	c	d	e	f	g	l	m
Ion current	2.900	2.300	0.070	−0.340	1.500	0.850	0.135	0.000	0.750
Electron current	2.900	2.300	0.110	−0.380	−2.800	5.100	0.135	2.800	0.650

while the charged particle density may be computed from the double-probe characteristic using

$$N = [H(\psi)]/[K_+\Theta_-^{1/2}(\beta + |\chi_*|)^\alpha] \quad (13)$$

where χ_* refers to χ_1 or χ_2, whichever is appropriate.

3. Probe Response in a Transient Plasma

The primary object of the transient analysis is to relate the mean-measured plasma properties and the measured instantaneous, average, and mean-square probe currents to the actual mean and statistical plasma properties. Because the steady-state measurements defined by Eqs. (1, 8, and 11) are taken to be instantaneously valid the transient analysis is in fact quasi-steady and will be limited by a critical frequency (determined by Berman and Lam[19] to be somewhat less than the ion plasma frequency) below which the steady-state probe theories remain useful.

Assume that the instantaneous descriptions of the probe signals are given by $J(t)$, $H(t)$, $\Phi_{wf}(t)$. Then as is customary in turbulence theory[20] the momentary value of the plasma parameters may be defined as

$$N(t) = \bar{N} + n(t),\ \Phi(t) = \bar{\Phi} + \phi(t)$$
$$\Theta_-(t) = \bar{\Theta}_- + \theta_-(t),\ \bar{\Theta}_+(t) = \bar{\Theta}_+ + \theta_+(t)$$

where the lower case letters denote the fluctuating components defined by

$$\langle n(t)\rangle = \langle \theta_-(t)\rangle = \langle \theta_+(t)\rangle = \langle \phi(t)\rangle = 0$$

and the bar or angular brackets $\langle\ \rangle$ denote a time average, the integration period being long with respect to the time scale of the turbulence, yet small compared to any slow variations in the plasma parameters which are not regarded as belonging to the turbulence. Similarly define the instantaneous single-probe current, double-probe current, and floating potential measurement by

$$J(t) = \bar{J} + j(t),\ H(t) = \bar{H} + h(t),\ \Phi_{wf}(t) = \Phi_{wf} + \phi_{wf}(t) \quad (14)$$

3.1 Single Probe Ion or Electron Current Collection

If Eq. (1) is expanded in a Taylor series then to first order the normalized instantaneous probe current is of the form

$$J/J_0 = 1 + F_N(n/\bar{N}) + F_{\Theta_-}(\theta_-/\bar{\Theta}_-) + F_{\Theta_+}(\theta_+/\bar{\Theta}_+) + F_\Phi(\phi/\bar{\Theta}) \quad (15)$$

where J_o is defined to be the current $J(\bar{N},\bar{\Theta}_-,\bar{\Theta}_+,\bar{\Phi})$ computed at the average properties. Now, the average measured probe current $\bar{J} = \langle J(N,\Theta_-,\Theta_+,\Phi)\rangle$ is not necessarily equal to the current J_o. If the series expansion is continued and the time average taken then the dependence of $\bar{J}/J_o$ on the fluctuating magnitudes is

$$\begin{aligned}\bar{J}/J_o = {} & 1 + \tfrac{1}{2}[F_{NN}\langle(n/\bar{N})^2\rangle + F_{\Theta_-\Theta_-}\langle(\theta_-/\bar{\Theta}_-)^2\rangle + \\ & F_{\Theta_+\Theta_+}\langle(\theta_+/\bar{\Theta}_+)^2\rangle + F_{\Phi\Phi}\langle(\phi/\bar{\Theta}_-)^2\rangle] + F_{N\Theta_-}\langle(n/\bar{N})(\theta_-\bar{\Theta}/_-)\rangle + \\ & F_{N\Theta_+}\langle(n/\bar{N})(\theta_+/\bar{\Theta}_+)\rangle + F_{N\Phi}\langle(n/\bar{N})(\phi/\bar{\Theta}_-)\rangle + \\ & F_{\Phi\Theta_-}\langle(\phi/\bar{\Theta}_-)(\theta_-/\bar{\Theta}_-)\rangle + F_{\Phi\Theta_+}\langle(\phi/\bar{\Theta}_-)(\theta_+/\bar{\Theta}_+)\rangle + \\ & F_{\Theta_+\Theta_-}\langle(\theta_+/\bar{\Theta}_+)(\theta_-/\bar{\Theta}_-)\rangle \end{aligned} \quad (16)$$

where terms up to and including those of second order are retained. The remaining measurable quantity, namely the mean-square current fluctuation, is found from Eqs. (14–16) to be

$$\begin{aligned}\langle(j/J_o)^2\rangle = {} & F_N^2\langle(n/\bar{N})^2\rangle + F_{\Theta_-}^2\langle(\theta_-/\bar{\Theta}_-)^2\rangle + \\ & F_{\Theta_+}^2\langle(\theta_+/\bar{\Theta}_+)\rangle + F_\Phi^2\langle(\phi/\bar{\Theta}_-)^2\rangle + \\ & 2[F_NF_{\Theta_+}\langle(n/\bar{N})(\theta_+/\bar{\Theta}_+)\rangle + F_NF_\Phi\langle(n/\bar{N})(\phi/\bar{\Theta}_-)\rangle + \\ & F_NF_{\Theta_-}\langle(n/\bar{N})(\theta_-/\bar{\Theta}_-)\rangle + F_\Phi F_{\Theta_-}\langle(\phi/\bar{\Theta}_-)(\theta_-/\bar{\Theta}_-)\rangle + \\ & F_\Phi F_{\Theta_+}\langle(\phi/\bar{\Theta}_-)(\theta_+/\bar{\Theta}_-)\rangle + F_{\Theta_-}F_{\Theta_+}\langle(\theta_-/\bar{\Theta}_-)(\theta_+/\bar{\Theta}_+)\rangle] \end{aligned} \quad (17)$$

where again terms up to and including those of second order are retained. If the fluctuations are truly random and large enough to require the inclusion of third order terms including the triple correlations $\langle(n)(\theta_-)(\phi)\rangle$, $\langle(n)(n)(\theta_-)\rangle$, then, for example, $\bar{J}/J_o$ would depend on thirty unknowns. The coefficients F_N, F_Θ, etc., evaluated at $\bar{N}$, $\bar{\Theta}_+$, $\bar{\Theta}_-$, $\bar{\Phi}$, are just the normalized partial derivatives appearing in the Taylor series expansion. For example,

$$F_N = (\bar{N}/J_o)(\partial J_o/\partial\bar{N}) = 1 - \frac{(\frac{1}{2})a}{[\ln(r/\lambda_o) + b]^2} \times$$
$$\ln(\beta_o + |\chi_o|) + [\alpha_o/(\beta_o + |\chi_o|)] \times$$
$$\left\{-\frac{(\frac{1}{2})l}{[\ln(r/\lambda_o)]^2} + \left[\frac{3}{2}g\left(\ln\frac{r}{\lambda_o}\right)^2 + \frac{\frac{1}{2}}{[\ln(r/\lambda_o)]^2}\right](\bar{\Theta}_+/\bar{\Theta}_-)\right\}$$

where the appropriate numerical constants may be selected from Table 1. Space alone prohibits the listing of the fourteen coefficients appearing in Eqs. (16) and (17); however they are given in Ref. 18.

The coefficients (F_N^2,$F_{\Theta_-}^2$, etc.) of the unknown correlation terms appearing in Eq. (17) depend on the controllable parameters r/λ_o and $\bar{\Phi}/\bar{\Theta}_-$. In principle a set of independent equations may therefore be formed whose solution will provide a first approximation for these correlation terms.

3.2 Floating Potential Measurement

A knowledge of the dependence of the normalized, instantaneous, mean and mean-square floating potential measurements on the mean and mean-square fluctuating values of the plasma parameters is of considerable practical interest as well as a prerequisite to understanding the double-probe transient response. The necessary results can be determined from Eq. (8) by applying the technique outlined in Sec. 3.1 and are included in Ref. 18.

3.3 Double-Probe Current Collection

The instantaneous double-probe current-voltage characteristic is defined by Eq. (11). However, a bias voltage applied between the probes results in an asymmetric division of the probe potentials around the floating potential and consideration of this effect is required in order to properly evaluate the Taylor series coefficients. Mathematically this is best illustrated by expressing the nondimensionalized potential $\psi = V/\Theta_- = \psi_1 + \psi_2$ as the sum of the individual probe potentials measured with respect to the floating potential. Thus

$$\chi_1 = \chi_f - \psi_1 \qquad \chi_2 = \chi_f + \psi_2 \quad (18)$$

and the double-probe ion current defined by Eq. (1) may be written as

$$J_{1,2}' = K_+N_+\Theta_-^{1/2}(\beta - \chi_f + \psi')^\alpha \quad (19)$$

where ψ' is either (ψ_1) or ($-\psi_2$), while Eq. (11) describing the double-probe response becomes

$$H = K_{2_+}N_+\Theta_-^{1/2}[(\beta - \chi_f - \psi_2)^\alpha - (\beta - \chi_f + \psi_1)^\alpha \exp(\psi)]\ [1 + (A_2/A_1)\exp(\psi)]^{-1} \quad (20)$$

Now the applied potential V is assumed constant with respect to the time scale of the turbulence and since the floating potential χ_f is independent of fluctuations in plasma potential, it follows that the double-probe currents defined by Eqs. (19) and (20) are only sensitive to fluctuations in N, Θ_-, and Θ_+. If the procedure described in Sec. 3.1 is applied to Eq. (20) while using the results discussed in Sec. 3.2 to describe current fluctuations driven by changes in floating potential (which results from fluctuations in plasma parameters other than the plasma potential) then the normalized instantaneous, mean, and mean-square double-probe currents are given by

$$H/H_o = 1 + G_N(n/\bar{N}) + G_{\Theta_-}(\theta_-/\bar{\Theta}_-) + G_{\Theta_+}(\theta_+/\bar{\Theta}_+) \quad (21)$$

$$\bar{H}/H_o = 1 + G_{N\Theta_-}\langle(n/\bar{N})(\theta_-/\bar{\Theta}_-)\rangle + G_{N\Theta_+}\langle(n/\bar{N})(\theta_+/\bar{\Theta}_+)\rangle + G_{\Theta_-\Theta_+}\langle(\theta_-/\bar{\Theta}_-)(\theta_+/\bar{\Theta}_+)\rangle + \tfrac{1}{2}[G_{NN}\langle(n/\bar{N})^2\rangle + G_{\Theta_-\Theta_-}\langle(\theta_-/\bar{\Theta}_-)^2\rangle + G_{\Theta_+\Theta_+}\langle(\theta_+/\bar{\Theta}_+)^2\rangle] \quad (22)$$

$$\langle(h/H_o)^2\rangle = G_N{}^2\langle(n/\bar{N})^2\rangle + G_{\Theta_-}{}^2\langle(\theta_-/\bar{\Theta}_-)^2\rangle + G_{\Theta_+}{}^2\langle(\theta_+/\bar{\Theta}_+)^2\rangle + 2[G_N G_{\Theta_-}\langle(n/\bar{N})(\theta_-/\bar{\Theta}_-)\rangle + G_N G_{\Theta_+}\langle(n/\bar{N})(\theta_+/\bar{\Theta}_+)\rangle + G_{\Theta_-}G_{\Theta_+}\langle(\theta_-/\bar{\Theta}_-)(\theta_+/\bar{\Theta}_+)\rangle] \quad (23)$$

where the coefficients (G_N, G_{Θ_-}, . . ., etc.), evaluated at $\bar{N}$, $\bar{\Theta}_+$, $\bar{\Theta}_-$, and ψ_o, are given in Ref. 18. Again these coefficients were found to be functions of the controllable parameters r/λ_o and ψ_o.

Equations (20) and (8) are valid in the OML (TSL) if α and β are replaced by the constants $\alpha = \frac{1}{2}$, $\beta = 1.0$ ($\alpha = 0$). Therefore if the appropriate values of α and β are used, the transient response of a double-probe operating in the OML ($r/\lambda < 1$) or TSL ($r/\lambda > 10^3$) may be determined from Eqs. (20) and (8) by using the technique outlined in Sec. 3.1. The results similar in form to Eqs. (21–23) are included in Ref. 18.

3.4 Electron Temperature Measurement

The double-probe is particularly well-suited to measuring the electron temperature in an unsteady plasma as its instantaneous current-voltage characteristic is independent of fluctuations in plasma potential. From Eqs. (19) and (20) the ratio of mean to mean-measured electron temperature is found to be

$$(\bar{\Theta}_-/\bar{\Theta}_{-m}) = [(\partial\bar{H}/\partial V)/(\partial H_o/\partial V)]_{V=o}[1/(\langle J'\rangle/J_o')]_{V=o} \times \{[1 + \alpha/(\beta + |\chi_f|)]_o/[1 + \alpha/(\beta + |\chi_f|)]_m\}$$

where $\langle J'\rangle/J_o'$ may be determined from Eq. (19) and is given in Ref. 18. Now $\bar{H} = H_o(\bar{H}/H_o)$, thus

$$\partial\bar{H}/\partial V = (\partial H_o/\partial V)(\bar{H}/H_o) + H_o(\partial/\partial V)(\bar{H}/H_o)$$

However, from Eq. (20) $(H_o)_{V=o} = O$, whereas $[(\partial/\partial V)(\bar{H}/H_o)]_{V=o}$ is finite, therefore

$$(\bar{\Theta}_-/\bar{\Theta}_{-m}) = \left\{1 + \frac{[\langle H\rangle/H_o - \langle J'\rangle/J_o']_{V=o}}{(\langle J'\rangle/J_o')_{V=o}}\right\} \times \{[1 + \alpha/(\beta + |\chi_f|)]_o/[1 + \alpha/(\beta + |\chi_f|)]_m\} \quad (24)$$

where α, β, and χ_f are to be computed from the mean or mean-measured plasma properties as designated.

Expressions analogous to Eq. (24) were derived by Demetriades and Doughman[7] for the ratio of mean electron temperature to its mean value measured from the single-probe characteristic. It is of interest to note that in the absence of fluctuations in (Θ_-) the measurement of the electron temperature from either the single-probe or double-probe characteristic is formally correct at least to second order.

3.5 Number Density Measurement

The number density is usually computed from the value of the saturation current measured at some large potential (χ_N or ψ_N), and the measured electron temperature. If $\bar{N}_m$ is defined to be of the form

$$\bar{N}_m = \bar{J}/\{\bar{\Theta}_{-m}{}^{1/2}K[(\beta + |\chi_N|)^\alpha]_m\} \quad (25)$$

then the ratio of mean to mean-measured density is

$$\bar{N}/\bar{N}_m = (\{[\langle H(\psi_N)\rangle]/[H_o(\psi_N)]\}(\bar{\Theta}_-/\bar{\Theta}_{-m})^{1/2})^{-1} \times \{[(\beta + |\chi_f| + |\psi_N'|)^\alpha]_m/[(\beta + |\chi_f| + |\psi_N'|)^\alpha]_o\} \quad (26)$$

$$\bar{N}/\bar{N}_m = (\{[\langle J(\chi_N)\rangle]/[J_o(\chi_N)]\}(\bar{\Theta}_-/\bar{\Theta}_{-m})^{1/2})^{-1} \times \{[(\beta + |\chi_N|)^\alpha]_m/[(\beta + |\chi_N|)^\alpha]_o\} \quad (27)$$

for computations from double-probe and single-probe characteristics, respectively.

3.6 Discussion

The equations presented in the previous sections relate the instantaneous, average and mean-square single-probe ion or electron currents (attracted by the probe potential) and the double-probe current to the instantaneous, average and mean-square plasma properties. In addition, the equations which relate the mean-meausured plasma properties to the actual mean plasma properties and their respective correlations are derived. In principle a set of independent equations can be formed from Eq. (17) or (23) using the controllable parameters r/λ_o, χ_o, and ψ_o. The solution of this set provides a first approximation for the unknown correlation terms to be used in the correct deduction of the mean plasma properties. In turn these mean properties may be used to improve the computation of the correlation terms, and so on.

Because of the large number of equations involved and since the coefficients (F_N, F_{Θ_-}, . . ., $G_{N\Theta_-}$, . . ., etc.) of the correlation terms are functions of three parameters (r/λ_o, $\bar{\Theta}_+/\bar{\Theta}_-$, χ_o, or ψ_o) a complete set of numerical values for these coefficients will not be presented; however, two general comments are in order. First, for moderately large values of the nondimensionalized potential (χ_o) the coefficients ($F_{N\Phi}$, F_Φ, etc.) of the terms involving fluctuations in plasma potential approach zero, even though the remaining coefficients (F_{NN}, F_{Θ_-}, etc.) are potential dependent. Therefore in this limit Eqs. (15–17) describing the single-probe current measurements become identical in form to Eqs. (21–23) describing the double-probe current measurements. Second, in the limit as the ratio of ion to electron temperature becomes small, all coefficients of the terms involving fluctuations in ion temperature become negligible and an exact evaluation of this temperature is not required. Therefore in the limit of moderately large χ_o and small $\bar{\Theta}_+/\bar{\Theta}_-$ the mean and mean-square single-probe and double-probe currents depend on the three correlation terms $\langle(n/\bar{N})^2\rangle$, $\langle(n/\bar{N})(\theta_-/\bar{\Theta}_-)\rangle$ and $\langle(\theta_-/\bar{\Theta}_-)^2\rangle$.

Some typical values for the various coefficients are included in Sec. 4 in order to make meaningful comparisons with experiments. Additional tables and graphs giving values for these coefficients are included in Ref. 18.

4. Experiment

The objective of the experiment is to verify the results put forth in Sec. 3 and to demonstrate the feasibility of using Langmuir probes as diagnostic tools to determine certain

Table 2 Flow parameters

	Temperature, °K		Density, cm^{-3}			Collision frequency	Debye	Mean-free-paths				
Flow	elect	ion, atom	elect, ion	atom	Mach no.	elect-ion, sec^{-1}	length, cm	charge exch.	atom-atom	electron-ion	electron-neutral	ion-ion
I	1750	~130	2×10^{11}	1×10^{14}	~9	6×10^7	8×10^{-4}	1.6	1.2	4×10^{-1}	3×10^2	8×10^{-4}
II	1500	~150	4×10^{11}	7×10^{13}	~8	1×10^8	5×10^{-4}	2.1	1.8	2×10^{-1}	4×10^2	6×10^{-4}

properties of unsteady plasmas. Both the mean and root-mean-square double-probe current and single-probe ion and electron currents are measured and the influence of the controllable parameters r/λ_o and $V/\bar{\Theta}_-$ on each of these measurements is demonstrated.

4.1 Plasma and Flowfield

The measurements are taken in an unsteady highly expanded low-density flowing argon plasma. Gas from a supply manifold is metered, then injected in any desired combination of axial, radial, or swirl directions, into the stagnation chamber where it is inductively heated using a 0–50 kw RF generator. It then expands through a 4.45-cm-diam free-jet orifice to provide a supersonic low-density flow at the test position from where it is removed by continuous pumping. The properties of this jet were established as described in Refs. 18 and 21 and are shown in Table 2 for flow conditions I and II.

The stability of the flow is dependent on the mass flow rate and method of injection. Typical power spectra taken at the test position showing fluctuations in total light intensity and probe saturation currents are shown in Fig. 1. This spectrum may be roughly approximated by the given exponential relation, the inverse transform of which implies a micro time scale corresponding to a length scale of 80 cm (based on $U \approx 1700$ m/sec) for the small-scale disturbances. Since the jet spans ~35 cm between stagnation chamber and Mach disk the flow is considered to be unsteady in time with the probe and phototube sampling those regions of plasma having different properties which are being continually convected by the test position. Because light intensity measurements are independent of changes in potential, fluctuations must be present in one or more of the other plasma properties. Consequently this plasma is well-suited to the study of the results presented in Sec. 3.

4.2 Probes and Their Application in an Unsteady Plasma

The cylindrical Langmuir probes are constructed by leaving a length of bare tungsten wire protruding from an insulated stem which is of sufficient length to insure that the collecting surface samples the undisturbed flow. The various probes and their geometries are listed in Table 3. Probes having the same diameter were used together as double-probes and individually as single-probes. Comparison of the information presented in Tables 2 and 3 indicates that all mean free paths with the exception of that for ion-ion collisions are large compared to the probe diameters. Detailed descriptions of the Langmuir probe construction, cleaning, circuitry and methods for obtaining data may be found in Ref. 18.

In the high Mach number flow described in Sec. 4.1, $(kT_-/M_+)^{1/2} < U \ll (kT/M_-)^{1/2}$, thus ion motion is primarily convective, and therefore dependent on yaw angle, while the electron motion is random. Because electron motion is independent of flow velocity the transient theory is obviously valid for electron collection. If transverse velocity fluctuations are not significant (their presence would not be expected in the time varying plasma studied here) then the probe can effectively be aligned with the flow and the transient theory is valid for ion collection as well.

An important consideration in any plasma experiment is the disturbance of the plasma by the presence of the probe. Because of the possible influence of transverse velocity fluctuations on ion current collection, electron current measurements were thought to be of particular importance. The area of a probe was therefore limited such that its rate of electron collection was small compared to the rate at which electrons are convected across the ~12-cm-diam near-isentropic flow region at the test position. For the largest probe electron current $[(J_-)\text{probe}/(J_-)\text{plasma} \sim 6 \times 10^{-2}$, while the disturbance was proportionally smaller for the other probes and negligible for ion current collection.

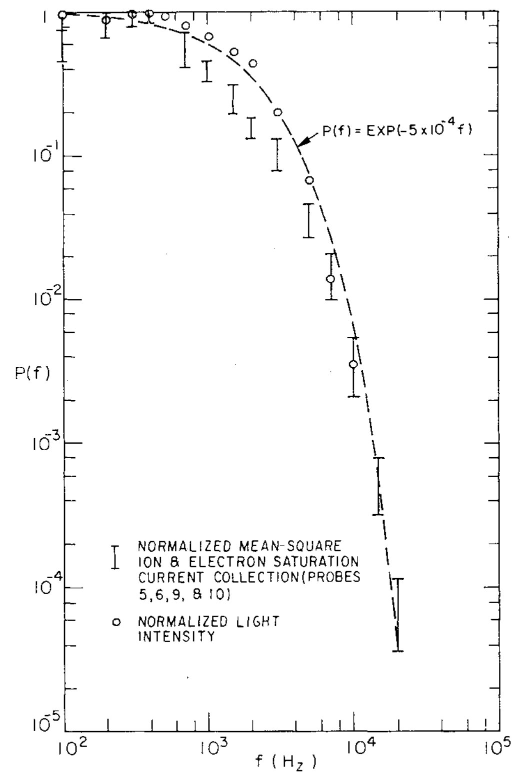

Fig. 1 Power spectrum.

Limiting the probe area to minimize the possibility of disturbing the plasma requires small L/r for the larger diameter probes and some compromise between the two effects is required. Measurements[21] indicated that the increase in current per unit length to the large diameter probes, which resulted from the decrease in L/r to 3, could be limited to ~25% if the tips of the probes were insulated. Such probes were used in this study; however, the effect on measurements of the normalized mean-square probe current should be much less pronounced.

4.3 Single-Probe Ion and Electron Current Fluctuations

The normalized mean-square electron current is shown in Fig. 2 as a function of r/λ_o for $|\chi_o| = 25$ and the test condition of $\bar{\Theta}_+/\bar{\Theta}_- \approx 0.1$. Included in this figure are the coefficients of the correlation terms appearing in the appropriate simplified form (see Fig. 2) of Eq. (17). The data for flow conditions I and II were normalized to the theoretical curve of $F_N{}^2$ by assuming $\langle (n/\bar{N})^2 \rangle = 0.09$ and 0.0625, respectively. Therefore deviations from this curve imply contributions to $\langle (j/J_o)^2 \rangle$ from terms other than $\langle (n/\bar{N})^2 \rangle$. Because the data conform closely to the curve $(F_N{}^2)$ for moderate values of r/λ_o and increase with decreasing r/λ_o as the OML is approached, it is thought that $\langle (n/\bar{N})^2 \rangle$ provides the major con-

Table 3 Langmuir probes

Probe no.	1	2	3	4	5	6	7	8	9	10	11	12	13
$2r$ (*mm*)	0.010	0.025	0.127	0.127	0.254	0.254	1.01	1.01	1.52	1.52	2.03	1.27	1.27
A (*mm*^{2})	0.243	0.465	1.65	1.65	2.38	2.48	6.90	6.98	13.4	13.4	15.1	11.5	11.5
L/r	1500	460	65.2	65.2	29.0	30.2	4.20	4.30	3.66	3.66	2.32	3.78	3.78

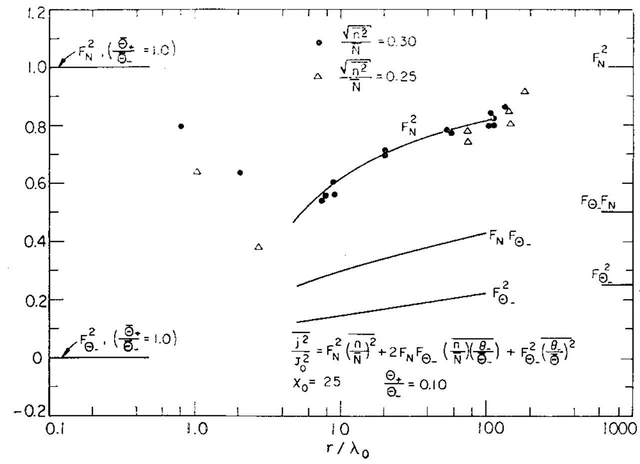

Fig. 2 Normalized mean-square saturation electron current as a function of r/λ_0.

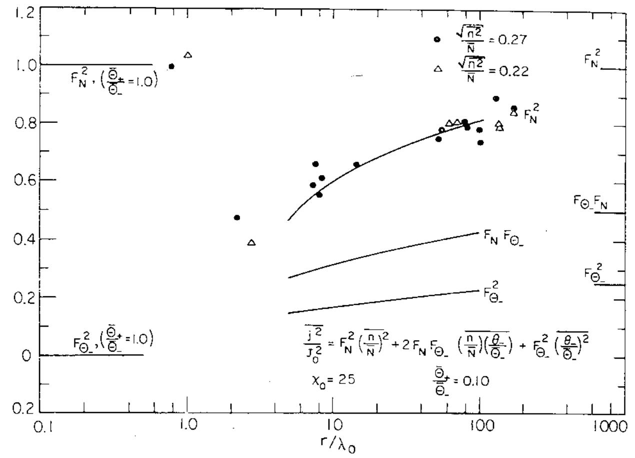

Fig. 4 Normalized mean-square saturation ion current as a function of r/λ_0.

tribution to $\langle (j/J_o)^2\rangle$. In principle the individual correlation terms may be determined directly from the data; however, the scatter is sufficient to prohibit an accurate evaluation using r/λ_o as the only adjustable parameter.

The dependence of the intensity of the fluctuations in electron current on $|\chi_o|$ is shown in Fig. 3, where the cumulative data from each probe were normalized to the theoretical curve of $F_N{}^2$ for the appropriate r/λ_o by assuming current fluctuations represent fluctuations in density only. Thus the data illustrate the potential dependence of the probe currents and do not reflect the data scatter as did Fig. 2. The data from probes 1 and 2 have not been included in Fig. 3 since these data lie as shown in Fig. 2 in the theoretically undefined region where $r/\lambda_o < 5$. Again deviations from the theoretical curves imply contributions to $\langle (j/J_o)^2\rangle$ from correlation terms other than $\langle (n/\bar{N})^2\rangle$. For values of $|\chi_o| > 20$ the normalized data conform quite well to the theoretical curves representing $\langle (n/\bar{N})^2\rangle$ vs $|\chi_o|$. The measurable fluctuations in probe current therefore result from fluctuations in density as $F_{\Theta-}{}^2$ and $F_N F_{\Theta-}$ remain nearly constant or increase with $|\chi_o|$.[18]

The preceding remarks also apply for the normalized mean-square ion current data shown in Figs. 4 and 5. Because the results obtained for ion and electron collection were consistent, it is concluded that ion collection was not appreciably disturbed by transverse velocity fluctuations and that electron collection did not significantly disturb the plasma.

4.4 Double-Probe Current Collection

Typical normalized mean-square double-probe data are shown in Fig. 6 where the cumulative data from each double-probe were normalized to the theoretical curve representing the contribution from number density fluctuations. Included in these figures are the coefficients of the correlation terms appearing in the appropriate simplified form (see Fig. 6) of Eq. (23). Recall from Figs. 2 and 4 that both F_N and F_Θ depend on a similar manner on r/λ_o, whereas the functional dependencies of G_N and G_Θ on $|\psi_o|$ are quite different. As a result, the individual correlation terms are more easily recovered from the double-probe data which is in effect more sensitive to fluctuations in electron temperature. Even so, the data again imply that fluctuations in charged particle density are responsible for the measured fluctuations in probe current.

Double-probe data are not presented for values of $|\psi_o| < 1.0$ for two reasons. First, there existed a residual noise level such that $\langle (j/\bar{J})^2\rangle$ becomes large as $\bar{J} \rightarrow 0$. Second, there was some drift in the potential of the double-probe system with time, and since it was necessary to take the r.m.s. and mean data separately to keep this residual noise level at a minimum, the common zero potential position could not be determined exactly. This uncertainty in potential introduces a significant error in $\langle (j/\bar{J})^2\rangle$ for small values of $|\psi_o|$. The effect was to increase the intensity indicated by one probe, while reducing that shown by the other probe.

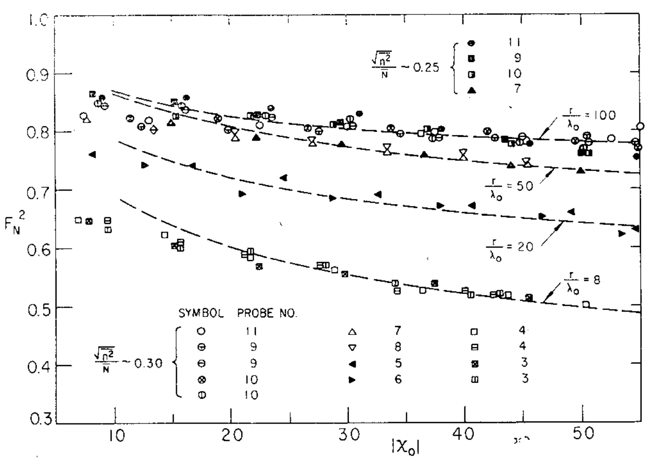

Fig. 3 Normalized mean-square saturation electron current as a function of $|\chi_0|$.

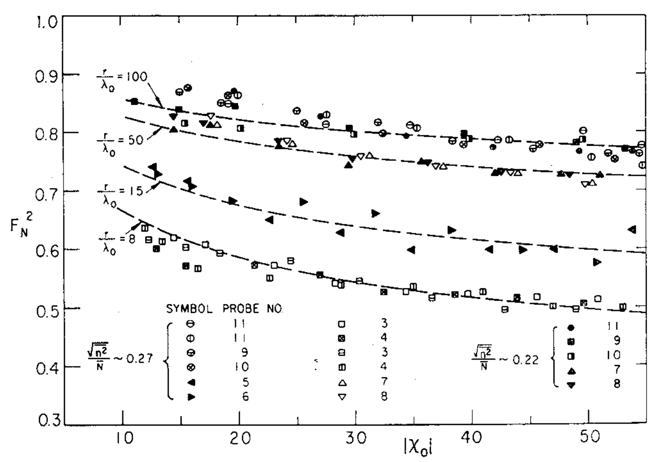

Fig. 5 Normalized mean-square saturation ion current as a function of $|\chi_0|$.

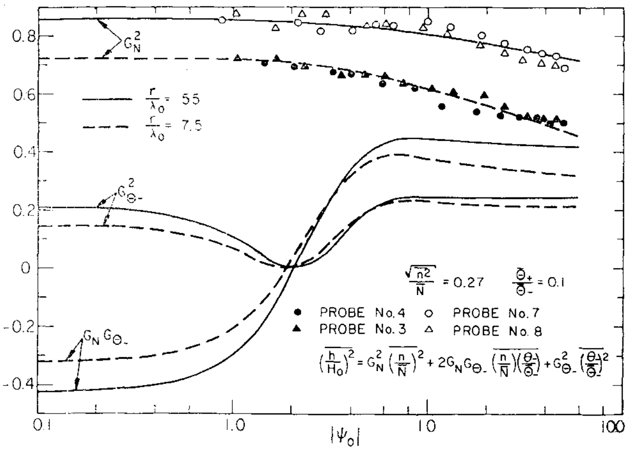

Fig. 6 Normalized mean-square current collection by double-probe.

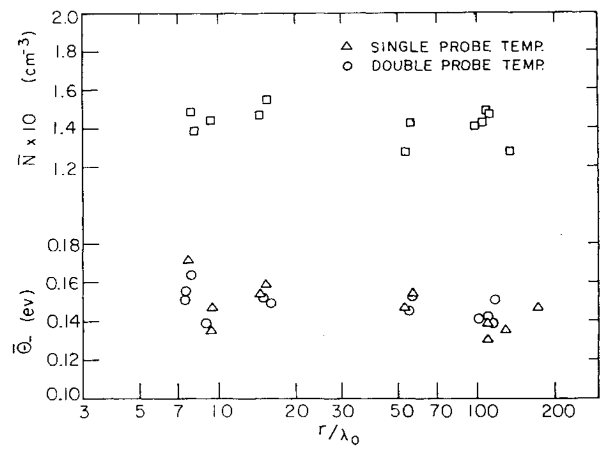

Fig. 8 Mean electron temperature and number density.

4.5 Relative Intensities of the Fluctuations

The first approximations for the individual correlation terms must be calculated from single-probe or double-probe data according to Eqs. (17) and (23), respectively. For $\bar{\Theta}_+/\bar{\Theta}_- \rightarrow O$, and large $|\chi_o|$ in the case of the single-probe, these equations reduce to the simplified forms shown in Figs. 4 and 6. If, as in this case, the relative intensities of the fluctuations are of moderate values, then Eqs. (16) and (22) reduce to $\bar{J}/J_o = \bar{H}/H_o = 1.0$ and the correlation terms may be calculated without iteration. The equations, in this simplified form, apply to all three independent measurements of current intensity reported in the previous sections. Therefore, consistent values for the correlation terms should be recovered from single-probe ion, single-probe electron, and double-probe current collection data.

Recall that the data imply (with the exception of fluctuations in potential) that fluctuations exist only in charged particle density. Therefore, the problem is again simplified and the relative intensity of these fluctuations may be determined from the data of each individual probe. The resulting values for flow condition I are shown in Fig. 7 for single-probe and double-probe data taken at values of the normalized potentials of $|\chi_o| = 25.0$ and $|\psi_o| = 2.0$, respectively. Electron current measurements indicated an average value for $\langle(n/\bar{N})^2\rangle$ of $\sim$0.30, while single-probe ion current and double-probe current measurements gave $\sim$0.27. The reason for this small difference was not apparent; however, data taken in flow II, not shown here, gave similar results of $\sim$0.25 and $\sim$0.22 for electron and ion collection, respectively.

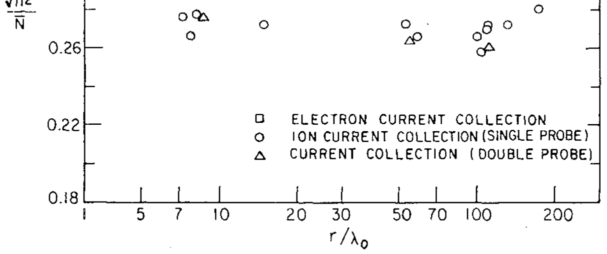

Fig. 7 Intensity of number density fluctuations.

4.6 Mean Plasma Properties

The mean electron temperatures as indicated by both single-probe and double-probe characteristics are shown in Fig. 8. Because $\bar{\Theta}_-/\bar{\Theta}_{-m} = 1.0$ in the absence of fluctuations in electron temperature these data were measured directly from the probe characteristics.

The mean charged particle number density, defined by Eqs. (26) or (27), was computed from both single-probe ion current and double-probe current measurements using values for the electron temperature determined by the respective methods. In the presence of moderate fluctuations in the plasma properties $\bar{J}(\chi_N)/J_o(\chi_N) = \bar{H}(\psi_N)/H_o(\psi_N) = 1.0$,[18] and since $\bar{\Theta}_-/\bar{\Theta}_{-m} = 1.0$, it follows that $\bar{N}/\bar{N}_m = 1.0$. Therefore the values of number density shown in Fig. 8 were computed without iteration from the probe characteristics.

The mean values of electron temperature and charged particle density shown in Fig. 8 were used in the computation of r/λ_o, χ_o, ψ_o and $\bar{\Theta}_+/\bar{\Theta}_-$. In turn, these parameters were used to compute the coefficients ($F_N, F_N\Theta_-, \ldots, G_N, G_N\Theta_-, \ldots$, etc.) of the correlation terms appearing in the equations defining the mean and mean-square current collection.

5. Conclusions

A theoretical and experimental investigation of the response of cylindrical free-molecule Langmuir probes to arbitrary (low-frequency) fluctuations in the plasma parameters N, Θ_-, Θ_+, and Φ was made and equations were derived which relate the instantaneous, average, and mean-square probe signals to the instantaneous, average, and mean-square plasma properties. In addition, the influence of fluctuations in the probe signals on measurements of electron temperature, charged particle density, and floating potential were theoretically and experimentally investigated.

The normalized probe current intensity was found to depend on the parameters r/λ_o, χ_o, ψ_o and $\bar{\Theta}_+/\bar{\Theta}_-$. In principle the controllable parameters r/λ_o, χ_o, and ψ_o could be used to form sets of independent equations from which individual and/or groups of correlation terms could be determined. Because independent sets of equations could be formed describing single-probe ion, single-probe electron, and double-probe current collections, three independent measurements of the correlation terms are possible.

In particular the theoretical dependencies of the mean and mean-square single-probe ion, single-probe electron, and double-probe currents on the controllable parameters r/λ_o, χ_o, and ψ_o were experimentally verified for the case $\bar{\Theta}_+/\bar{\Theta}_- \ll 1$. The individual correlation terms were evaluated and consistent results were obtained from the three independent current measurements. It was found that the individual correlation terms could be most easily determined from the double-probe response, since the mean-square double-probe current was most sensitive to changes in the controllable parameters. The mean-square probe current was found to be most sensitive to fluctiations in number density. Thus when data exhibit typical experimental scatter, $\langle(\theta_-/\bar{\Theta}_-)^2\rangle > \langle(n/N)^2\rangle$ if terms involving fluctuations in electron temperature are to be determined with accuracy.

The ratios of mean to mean-measured charged particle density and mean to mean-measured electron temperature were found to depend on the magnitudes of the individual correlation terms. However, for moderate values of the relative intensities the errors introduced by assuming $\bar{\Theta}_-/\bar{\Theta}_{-m} = \bar{N}/\bar{N}_m = 1.0$ were small compared to the typical data scatter.

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